Einstein-Podolsky-Rosen correlation seen from moving observers

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Abstract

Within the framework of relativistic quantum theory, we consider the Einstein-Podolsky-Rosen (EPR) gedanken-experiment in which measurements of the spin are performed by moving observers. We find that the perfect anti-correlation in the same direction between the EPR pair no longer holds in the observers' frame. This does not imply a breakdown of the non-local correlation. We explicitly show that the observers must measure the spin in appropriately chosen *different* directions in order to observe the perfect anti-correlation. This fact should be taken into account in utilizing the entangled state in quantum communication by moving observers.

In 1935, Einstein, Podolsky, and Rosen (EPR) [1] proposed a gedanken-experiment in an attempt to show that the description of physical reality by quantum theory is not complete. A variant EPR gedankenexperiment was put forth by Bohm [2] in which a pair of spin-1/2 particles with total spin zero are moving in opposite directions. This state has a remarkable property, known as the EPR correlation, that measurements of the spin show perfect anti-correlation in whichever direction and at however remote places they are performed. In this Letter, we formulate Bohm's version of the EPR correlation within the framework of relativistic quantum theory and consider a situation in which measurements are performed by moving observers. We here focus on the role played by the moving observers from the viewpoint of the unitary transformation of the spin under the Lorentz transformation [3]. We find that the perfect anti-correlation in the *same* direction between the spins of an EPR pair deteriorates in the moving observers' frame in the following sense: even if the observers measure the spins in the same direction in their frame (which is also the same in the laboratory frame), the results of measurements are not always anti-correlated. That is, the perfect anti-correlation in the *same* direction is not Lorentz invariant. The perfect entanglement should, however, be preserved since the Lorentz transformation is a local unitary operation [4]. The perfect anti-correlation is thus maintained in *different* directions. We then show that the degree of the violation of Bell's inequality [5, 6] decreases with increasing the velocity of the observers. This also is a consequence of a local unitary operator associated with the Lorentz transformation which does not imply a breakdown of non-local correlations. Our aim is to explore effects of the relative motion between the sender and receiver in quantum communication.

Consider a massive particle with mass M. In the rest frame of the particle, the four-momentum is given by the rest momentum $k^\mu = (Mc, 0, 0, 0)$. In this frame, the state $|k, \sigma\rangle$ is specified in terms of the eigenvalues of the Hamiltonian H, the momentum operator $\vec{P}$, and the z-component of the total angular momentum operator $\vec{J}$ as $H|k,\sigma\rangle = Mc^2|k,\sigma\rangle$, $\vec{P}|k,\sigma\rangle = 0$, and $J^3|k,\sigma\rangle = \sigma\hbar|k,\sigma\rangle$, respectively. Since the momentum k^μ is invariant under the spatial rotation group $SO(3)$, a rotation $R^\mu{}_\nu$ is represented by a $(2j+1)$-dimensional unitary matrix $D^{(j)}(R)$,

$$U(R)\,|k,\sigma\rangle = \sum_{\sigma'} D^{(j)}_{\sigma'\sigma}(R)\,|k,\sigma'\rangle, \tag{1}$$

where j is an integer or a half-integer and $-j \le \sigma \le j$. Note that j is the spin of the particle and σ its z-component because the orbital angular momentum is absent in the rest frame of the particle.

In the laboratory frame, the four-momentum of the particle has a generic form $p^\mu = (\sqrt{|\vec{p}|^2 + M^2c^2}, p^1, p^2, p^3)$, which is obtained by performing a standard Lorentz transformation $L(p)^\mu{}_\nu$ on the rest momentum k^μ, i.e. $p^\mu = L(p)^\mu{}_\nu k^\nu$, where $\mu, \nu = 0, 1, 2, 3$ and repeated indices are assumed to be summed. An explicit form of $L(p)^\mu{}_\nu$ is written as

$$\begin{aligned} L(p)^0{}_0 &= \gamma, \qquad L(p)^0{}_i = L(p)^i{}_0 = p^i/Mc, \\ L(p)^i{}_k &= \delta_{ik} + (\gamma - 1)\, p^i\, p^k/|\vec{p}|^2, \end{aligned} \tag{2}$$

where $\gamma = \sqrt{|\vec{p}|^2 + M^2c^2}/Mc$ and $i, k = 1, 2, 3$. Using the unitary operator corresponding to $L(p)^\mu{}_\nu$, the state in this frame is defined by $|p,\sigma\rangle \equiv U(L(p))\,|k,\sigma\rangle$.

Consider now a situation in which an observer is moving with respect to the laboratory frame, and let $\Lambda^\mu{}_\nu$ be the corresponding Lorentz transformation. For this observer, the state of the particle is described by $U(\Lambda)\,|p,\sigma\rangle = U(L(\Lambda p))\,U(W(\Lambda, p))\,|k,\sigma\rangle$, where

$$W(\Lambda, p)^\mu{}_\nu = \left[L^{-1}(\Lambda p)\,\Lambda\,L(p)\right]^\mu{}_\nu \tag{3}$$

is the Wigner rotation [7]. The Wigner rotation is an element of the spatial rotation group $SO(3)$ since it leaves the rest momentum k^μ unchanged by the definition of $L(p)^\mu{}_\nu$. It follows then from Eq. (1) that

$$U(\Lambda)\,|p,\sigma\rangle = \sum_{\sigma'} D^{(j)}_{\sigma'\sigma}(W(\Lambda,p))\,|\Lambda p,\sigma'\rangle. \tag{4}$$

To be specific, suppose that a massive spin-1/2 particle (e.g. electron) is moving along the x-axis with the laboratory-frame four-momentum given by $p^\mu = (Mc\cosh\xi, Mc\sinh\xi, 0, 0)$, where the rapidity ξ is related to the velocity of the particle v by $v/c = \tanh\xi$. In this case, the Lorentz transformation (2) becomes

$$L(p)^\mu{}_\nu = \begin{pmatrix} \cosh\xi & \sinh\xi & 0 & 0 \\ \sinh\xi & \cosh\xi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

Suppose that an observer is moving along the z-axis with the velocity given by $V = c\tanh\chi$. The corresponding Lorentz transformation reads

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \cosh\chi & 0 & 0 & -\sinh\chi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh\chi & 0 & 0 & \cosh\chi \end{pmatrix}. \tag{6}$$

The Wigner rotation (3) is then reduced to a rotation about the y-axis. The angle δ of this rotation is given by

$$\tan\delta = \frac{\sinh\xi\sinh\chi}{\cosh\xi + \cosh\chi}. \tag{7}$$

This angle δ becomes $\xi\chi/2$ in the limit of $\xi \to 0$ and $\chi \to 0$, and $\pi/2$ in the limit of $\xi \to \infty$ and $\chi \to \infty$. If either $\xi = 0$ or $\chi = 0$, δ vanishes. For the case of the spin-1/2 particle, rotations are represented by the Pauli matrices. Using the Pauli matrix σ_y, the transformation law (4) thus becomes

$$\begin{aligned} U(\Lambda)\,|p,\uparrow\rangle &= \cos\frac{\delta}{2}\,|\Lambda p,\uparrow\rangle + \sin\frac{\delta}{2}\,|\Lambda p,\downarrow\rangle, && (8) \\ U(\Lambda)\,|p,\downarrow\rangle &= -\sin\frac{\delta}{2}\,|\Lambda p,\uparrow\rangle + \cos\frac{\delta}{2}\,|\Lambda p,\downarrow\rangle, && (9) \end{aligned}$$

where $\uparrow = +1/2$ and $\downarrow = -1/2$. That is, the spin is rotated about the y-axis through the angle δ in the observer's frame.

A physical picture of this spin rotation is as follows. The Lorentz transformation $\Lambda^{\mu}{}_{\nu}$ "rotates" the direction of the momentum from p^{μ} to Λp^{μ}. The spin is simultaneously rotated by this Lorentz transformation since the spin is coupled with the momentum in relativistic quantum theory. In non-relativistic quantum theory, the Galilean transformation rotates the direction of the momentum but not the spin. Note that the angle of rotation for the spin is *not* equal to that for the momentum because $\Lambda^{\mu}{}_{\nu}$ is not a spatial rotation but a Lorentz transformation. The rotation of the spin comes from the fact that $\Lambda L(p)^{\mu}{}_{\nu}$ and $L(\Lambda p)^{\mu}{}_{\nu}$ are not equal even though both of them bring the momentum k^{μ} to Λp^{μ}.

We use the transformation law obtained above to analyze the relativistic EPR correlation. Suppose that a pair of spin-1/2 particles with total spin zero are moving away from each other in the x direction. This situation is described by the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[\,|p_+,\uparrow\rangle|p_-,\downarrow\rangle - |p_+,\downarrow\rangle|p_-,\uparrow\rangle\,\Big], \tag{10}$$

where $p^{\mu}_{\pm} = (Mc\cosh\xi, \pm Mc\sinh\xi, 0, 0)$. Unlike the non-relativistic case, we need to explicitly specify the motion of the particles because the Lorentz transformation of the spin depends on the momentum as in Eq. (4). In the EPR experiment, we have two observers who perform measurements on the particles, respectively. Here we assume that both observers are moving in the z direction at the same velocity V. Using the transformation formulas (8) and (9), we find that the moving observers see the state (10) as

$$\begin{aligned} U(\Lambda)|\psi\rangle \;=\; & \frac{1}{\sqrt{2}}\Big[\cos\delta\Big(|\Lambda p_+,\uparrow\rangle|\Lambda p_-,\downarrow\rangle - |\Lambda p_+,\downarrow\rangle|\Lambda p_-,\uparrow\rangle\Big) \\ & + \sin\delta\Big(|\Lambda p_+,\uparrow\rangle|\Lambda p_-,\uparrow\rangle + |\Lambda p_+,\downarrow\rangle|\Lambda p_-,\downarrow\rangle\Big)\Big], \end{aligned} \tag{11}$$

where δ is given by Eq. (7). Note that the spins of the two particles are rotated in opposite directions because they are moving oppositely.

From Eq. (11), we find that the measurements of the spin z-component will no longer show perfect anti-correlation. Note that the two observers are at rest in the common frame, since they are moving in the same direction at the same velocity with respect to the laboratory frame. The directions that are the same in this observers' frame are also the same in the laboratory frame. Thus, in non-relativistic theory, the measurements in the same direction of the observers' frame must be perfectly anti-correlated. Moreover, the z direction in the observers' frame is identical to that in the laboratory frame since the observers are moving along the z-axis. Nevertheless, the anti-correlation in the z direction is reduced in relativistic theory. (On the other hand, the measurements of the spin y-component are perfectly anti-correlated for any ξ and χ.) Note that in Eq. (11) the spin-singlet state is mixed with the spin-triplet state. This is because while the spin-singlet state is invariant under spatial rotations, it is not invariant under Lorentz transformations. That is, the Poincaré group $ISO(1,3)$ is larger than the spatial rotation group $SO(3)$.

Let us now examine Bell's inequality in the same situation [8]. Let Q and R be operators on the first particle corresponding to the spin z- and y- components, respectively. Similarly, let S and T be operators on the second particle corresponding to the spin component in the directions $(0,-1/\sqrt{2},-1/\sqrt{2})$ and $(0,-1/\sqrt{2},1/\sqrt{2})$, respectively. Then, for the state (11), we obtain

$$\langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle = 2\sqrt{2}\cos^2\delta. \tag{12}$$

The right-hand side decreases with increasing the velocity of the observers and with increasing that of the particles, and vanishes in the limit of $\xi \to \infty$ and $\chi \to \infty$.

Does this result imply a breakdown of the EPR correlation or a revival of the local realism? The answer is, of course, no. If the directions of measurements are rotated about the y-axis through δ for the first particle and through $-\delta$ for the second in accordance with the spin rotation, the measurements of the spin are perfectly anti-correlated and Bell's inequality remains maximally violated. While the perfect anti-correlation in the *same* direction no longer holds, the perfect anti-correlation is maintained in the appropriately chosen different directions. Naive measurements lead to wrong conclusions.

In conclusion, in the relativistic EPR experiment with a pair of massive spin-1/2 particles, the spin-singlet state is mixed with the spin-triplet state if the measurements are performed by orthogonally moving observers. This is because the Poincaré group is larger than the spatial rotation group. Therefore, the perfect anti-correlation in the same direction deteriorates. We must carefully choose the directions of measurements to obtain the perfect anti-correlation and the maximal violation of Bell's inequality which are utilized in quantum communication [9, 10, 11]. We can also obtain a similar conclusion in the case of massless particles.

After this manuscript was prepared, the authors become aware of the work by Alsing and Milburn [4]. Although they have considered a similar situation, there are essential differences from ours. They have discussed the Lorentz invariance of entanglement using the spin-triplet state. Here we have discussed the change of the anti-correlation due to the Lorentz transformation, using the spin-singlet state. Note that the entanglement is independent of the basis but the correlation depends on the basis to measure, and the spin-triplet state does not have the property of the anti-correlation in all the directions, unlike the spin-singlet state.

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References

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[2] D. Bohm, in *Quantum Theory* (Dover, New York, 1989), pp. 611–623.

[3] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 1995), chap. 2.5.

[4] P. M. Alsing and G. J. Milburn, Quantum Information and Computation **2**, 487 (2002).

[5] J. S. Bell, Physics **1**, 195 (1964).

[6] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

[7] E. P. Wigner, Ann. Math. **40**, 149 (1939).

[8] In a situation where the two particles are co-moving, Bell's inequality has been discussed using a different formulation by M. Czachor, Phys. Rev. A **55**, 72 (1997).

[9] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).

[10] C. H. Bennett, G. Brassard, and N. D. Mermin, Phys. Rev. Lett. **68**, 557 (1992).

[11] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).